New version of quantum mechanics at finite temperatures as a ground for description of nearly perfect fluids

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15.06.2010

Abstract

We suggest a more general than quantum statistical mechanics (*QSM*) microdescription of objects in a heat bath taken into account a vacuum as an object environment - modification of quantum mechanics at finite temperatures; we call it $(\hbar, k)$-dynamics ($\hbar kD$). This approach allows us in a new manner to calculate some important macroparameters and to modify standard thermodynamics. We create an effective apparatus for features description of nearly perfect fluids in various mediums. As an essentially new model of an object environment we suppose a quantum heat bath and its properties, including cases of cold and warm vacuums, are studied. We describe the thermal equilibrium state in place of the traditional density operator in term of a wave function the amplitude and phase of which have temperature dependence.

We introduce a new generative operator, Schroedingerian, or stochastic action operator, and show its fundamental role in the microdescription. We demonstrate that a new macroparameter, namely the effective action , can be obtained through averaging of the Schroedingerian over the temperature dependent wave function. It is established that such different parameters as internal energy, effective temperature, and effective entropy and their fluctuations can be expressed through a single quantity - the effective action.

PACS numbers: 05.40.-a, 03.75.Hh

1 Fundamental microtheory: $(\hbar, k)$- dynamics

We develop a version of the universal theory that is unlike to the particular theory proposed earlier for explaining of ratio ”shear viscosity to specific entropy’ in quark-gluon plasma. We propose to formulate a quantum-thermal dynamics or, briefly, $(\hbar, k)$- dynamics ($\hbar kD$), as a modification of standard quantum mechanics taking thermal effects into account. The principal distinction of such a theory from QSM is that in it the state of a microobject under the conditions of contact with the quantum heat bath (QHB) is generally described not by the density matrix but by a temperature-dependent complex wave function.

We note that this is not a ”thechnical sleight-of-hand”. Using the wave function, we thereby suppose to consider pure and mixt states simultaneously in the frame of Gibbs’ ensemble. It is in principle differs from Boltzmann’ assembly used in QSM.

A general idea of our investigation: to construct a theory it is necessary

1. to change $\hat{\rho}(T) \Rightarrow \Psi_T(q)$;

2. to introduce (except of Hamiltonian) a new operator - the stochastic action operator $\hat{j}$;

3. to use an idea of heat bath at $T = 0$ (”cold” heat bath) also;

4. to use an idea of vacuum at $T > 0$ (”thermal” vacuum) also.

This theory is based on a new microparameter, namely, the stochastic action operator. In this case, we demonstrate that averaging the corresponding microparameters over the temperature-dependent wave function, we can find the most important effective macroparameters, including internal energy, temperature, and entropy. They have the physical meaning of the standard thermodynamic quantities using in the phenomenological macrodescription.

1.1 The model of the QHB: a case of the ”cold” vacuum

To describe the environment with the holistic stochastic action that was previously called the thermal field vacuum by Umezawa, we introduce a concrete model, the QHB. According to this, the QHB is a set of weakly coupled quantum oscillators with all possible frequencies. The equilibrium thermal radiation can serve as a preimage of such a model in nature.

The specific feature of our understanding of this model is that we assume that we must apply it to both the ”thermal” ($T \neq 0$) and the ”cold” ($T = 0$) vacua. Thus, in the sense of Einstein, we proceed from a more general understanding of the thermal equilibrium, which can, in principle, be established for any type of

environmental stochastic action (purely quantum, quantum-thermal, and purely thermal).

We begin our presentation by studying the ”cold” vacuum and discussing the description of a single quantum oscillator from the number of oscillators forming the QHB model for $T = 0$ from a new standpoint.

But we recall that the lowest state in the energetic ($\Psi_n(q)$) and coherent states (CS) is the same. In the occupation number representation, the ”cold” vacuum in which the number of particles is $n = 0$ corresponds to this state. In the q representation, the same ground state of the quantum oscillator is in turn described by the real wave function

$$\Psi_0(q) = [2\pi(\Delta q_0)^2]^{-1/4} \exp\left\{-\frac{q^2}{4(\Delta q_0)^2}\right\}. \tag{1}$$

As is well known, CS are the eigenstates of the non-Hermitian particle annihilation operator $\hat{a}$ with complex eigenvalues. But they include one isolated state $|0_a\rangle = |\Psi_0(q)\rangle$ of the particle vacuum in which eigenvalue of $\hat{a}$ is zero

$$\hat{a}|0_a\rangle = 0|0_a\rangle; \quad \hat{a}|\Psi_0(q)\rangle = 0|\Psi_0(q)\rangle. \tag{2}$$

In what follows, it is convenient to describe the QHB in the q representation. Therefore, we express the annihilation operator $\hat{a}$ and the creation operator $\hat{a}^\dagger$ in terms of the operators $\hat{p}$ and $\hat{q}$ using the traditional method. We have

$$\hat{a} = \frac{1}{2}\left(\frac{\hat{p}}{\sqrt{\Delta p_0^2}} - i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}\right); \quad \hat{a}^\dagger = \frac{1}{2}\left(\frac{\hat{p}}{\sqrt{\Delta p_0^2}} + i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}\right). \tag{3}$$

The particle number operator then becomes

$$\hat{N}_a = \hat{a}^\dagger\hat{a} = \left(\frac{\hat{p}^2}{\Delta p_0^2} - \frac{1}{2}\hat{I} + \frac{\hat{q}^2}{\Delta q_0^2}\right) = \frac{1}{\hbar\omega}\left(\frac{\hat{p}^2}{2m} - \frac{\hbar\omega}{2}\hat{I} + \frac{m\omega^2\hat{q}^2}{2}\right). \tag{4}$$

The sum of the first and third terms in the parentheses forms the Hamiltonian $\mathscr{H}$ of the quantum oscillator, and after multiplying relation (7) by $\hbar\omega$ on the left and on the right, we obtain the standard interrelation between the expressions for the Hamiltonian in the $q-$ and $n-$ representations:

$$\hat{\mathscr{H}} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{q}^2}{2} = \hbar\omega(\hat{N}_a + \frac{1}{2}\hat{I}), \tag{5}$$

where $\hat{I}$ is the unit operator.

From the thermodynamics standpoint, we are concerned with the internal energy of the quantum oscillator in equilibrium with the ”cold” QHB. Its value is equal to the mean of the Hamiltonian calculated over the state $|0_a\rangle \equiv |\Psi_0(q)\rangle$:

$$U_0 = \langle\Psi_0(q)|\hat{\mathscr{H}}|\Psi_0(q)\rangle = \hbar\omega\langle\Psi_0(q)|\hat{N}_a|\Psi_0(q)\rangle + \frac{\hbar\omega}{2} = \frac{\hbar\omega}{2} = \varepsilon_0. \quad (6)$$

It follows from formula (9) that in the given case, the state without particles coincides with the state of the Hamiltonian with the minimum energy ε_0. The quantity ε_0, traditionally treated as the energy of zero oscillations, takes the physical meaning of the internal energy U_0 of the quantum oscillator in equilibrium with the ”cold” vacuum.

1.2 Passage to the ”thermal” vacuum

We can pass from the ”cold” to the ”thermal” vacuum in the spirit of Umezawa using the Bogoliubov $(u,v)-$ transformation with the temperature-dependent coefficients

$$u = \left(\frac{1}{2}\coth\frac{\hbar\omega}{2k_BT} + \frac{1}{2}\right)^{1/2} e^{i\frac{\pi}{4}}; \quad v = \left(\frac{1}{2}\coth\frac{\hbar\omega}{2k_BT} - \frac{1}{2}\right)^{1/2} e^{-i\frac{\pi}{4}}. \quad (7)$$

In the given case, this transformation is canonical but leads to a unitarily nonequivalent representation because the QHB at any temperature is a system with an infinitely number of freedom degrees.

In the end, such a transformation reduces to passing from the set of quantum oscillator CS to a more general set of states called the thermal correlated coherent states (TCCS). They are selected because they ensure that the Schrodinger coordinate-momentum uncertainty relation is saturated at any temperature. From the of the second-quantization apparatus standpoint, the Bogoliubov $(u,v)-$ transformation ensures the passage from the original system of particles with the ”cold” vacuum $|0_a\rangle$ to the system of quasiparticles described by the annihilation operator $\hat{b}$ and the creation operator $\hat{b}^\dagger$ with the ”thermal” vacuum $|0_b\rangle$.

To obtain from ”cold” vacuum to ”thermal” one using (u,v) - Bogolubov’s transformations it is necessary to pass:

1. from CS to TCCS: $\Psi_0(q) \Rightarrow \Psi_T(q)$; $\quad |0_a\rangle \Rightarrow ||0_b\rangle$; 2. from particles to quasiparticles: $\hat{a} \Rightarrow \hat{b} = \hat{b}(T)$.

In this case, the choice of transformation coefficients (10) is fixed by the requirement that for any method of description, the expression for the mean energy of the quantum oscillator in thermal equilibrium be defined by the Planck formula, which can be obtained from experiments:

$$\mathscr{E}_{Pl.} = \hbar\omega(\exp\frac{\hbar\omega}{k_B T} - 1)^{-1} + \frac{\hbar\omega}{2} = \frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2k_B T}. \tag{8}$$

Earlier was shown by us, the state of the ”thermal” vacuum $|0_b\rangle \equiv |\Psi_T(q)\rangle$ in the $q-$ representation corresponds to the complex wave function

$$\Psi_T(q) = [2\pi(\Delta q)^2]^{-1/4}\exp\left\{-\frac{q^2}{4(\Delta q)^2}(1 - i\alpha)\right\}, \tag{9}$$

where

$$(\Delta q)^2 = \frac{\hbar}{2m\omega}\coth\frac{\hbar\omega}{2k_B T}; \quad \alpha = \left[\sinh\frac{\hbar\omega}{2k_B T}\right]^{-1}; \quad (\Delta p)^2 = \frac{\hbar m\omega}{2}\coth\frac{\hbar\omega}{2k_B T}. \tag{10}$$

We note that the expressions for the probability densities $\rho_T(q)$ and $\rho_T(p)$ have already been obtained by Bloch, but the expressions for the phase that depend on the parameter α play a very significant role and were not previously known. It is also easy to see that as $T \to 0$, the parameter $\alpha \to 0$ and the function $\Psi_T(q)$ from the set of TCCS passes to the function $\Psi_0(q)$ from the set of CS.

Of course, the states from the set of TCCS are the eigenstates of the non-Hermitian quasiparticle annihilation operator $\hat{b}$ with complex eigenvalues. They also include one isolated state of the quasiparticle vacuum in which eigenvalue of $\hat{b}$ is zero,

$$\hat{b}|0_b\rangle = 0|0_b\rangle; \quad \hat{b}|\Psi_T(q)\rangle = 0|\Psi_T(q)\rangle. \tag{11}$$

Using condition (15) and expression (12) for the wave function of the ”thermal” vacuum, we obtain the expression for the operator $\hat{b}$ in the $q-$ representation:

$$\hat{b} = \frac{1}{2}\sqrt{\coth\frac{\hbar\omega}{2k_B T}}\left[\frac{\hat{p}}{\sqrt{\Delta p_0^2}} - i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}(\coth\frac{\hbar\omega}{2k_B T})^{-1}(1 - i\alpha)\right]. \tag{12}$$

The corresponding quasiparticle creation operator has the form

$$\hat{b}^{\dagger} = \frac{1}{2}\sqrt{\coth\frac{\hbar\omega}{2k_BT}}\left[\frac{\hat{p}}{\sqrt{\Delta p_0^2}} + i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}(\coth\frac{\hbar\omega}{2k_BT})^{-1}(1+i\alpha)\right]. \tag{13}$$

We can verify that as $T \to 0$, the operators $\hat{b}^{\dagger}$ and $\hat{b}$ for quasiparticles pass to the operators $a^{\dagger}$ and $\hat{a}$ for particles and $|0_b\rangle \Rightarrow |0_a\rangle; \quad \Psi_T(q) \Rightarrow \Psi_0(q)$.

Acting just as above, we obtain the expression for the quasiparticle number operator in the $q-$ representation

$$\hat{N}_b = \hat{b}^{\dagger}\hat{b} = \frac{1}{4}\coth\frac{\hbar\omega}{2k_BT}\left[\frac{\hat{p}^2}{\Delta p_0^2} - 2(\coth\frac{\hbar\omega}{2k_BT})^{-1}(\hat{I} + \frac{\alpha}{\hbar}\{\hat{p},\hat{q}\}) + \frac{\hat{q}^2}{\Delta q_0^2}\right], \tag{14}$$

where we take $1+\alpha^2 = \coth^2\frac{\hbar\omega}{2k_BT}$ into account when calculating the last term.

1.3 Hamiltonian in TCCS

Passing from the quasiparticle number operator to the original Hamiltonian and multiplying by $\hbar\omega$, we obtain

$$\hat{\mathscr{H}} = \hbar\omega(\coth\frac{\hbar\omega}{2k_BT})^{-1}\left[\hat{N}_b + \frac{1}{2}(\hat{I} + \frac{\alpha}{\hbar}\{\hat{p},\hat{q}\})\right]. \tag{15}$$

We stress that the operator $\{\hat{p},\hat{q}\}$ in formula (19) can also be expressed in terms of bilinear combinations of the operators $\hat{b}^{\dagger}$ and $\hat{b}$, but they differ from the quasiparticle number operator N_b. This means that the operators $\hat{\mathscr{H}}$ and $\hat{N}_b$ do not commute and that the wave function of form (12) characterizing the state of the "thermal" vacuum is therefore not the eigenfunction of the Hamiltonian.

As before, we are interested in the thermodynamic quantity, namely, the internal energy U of the quantum oscillator now in thermal equilibrium with the "thermal" QHB. Calculating it just as earlier, we obtain

$$U = \hbar\omega(\coth\frac{\hbar\omega}{2k_BT})^{-1}\left[\langle\Psi_T(q)|\hat{N}_b|\Psi_T(q)\rangle + \frac{1}{2} + \frac{\alpha}{2\hbar}\langle\Psi_T(q)|\{\hat{p},\hat{q}\}|\Psi_T(q)\rangle\right] \tag{16}$$

in the $q-$ representation. Because we averaging over the quasiparticle vacuum in formula (20), the first term in it vanishes. At the same time, it was shown earlier by us that

$\langle\Psi_T(q)|\{\hat{p},\hat{q}\}|\Psi_T(q)\rangle = \hbar\alpha.$

As a result, we obtain the expression for the internal energy of the quantum oscillator in the ”thermal” QHB in the $\hbar kD$:

$$U = \frac{\hbar\omega}{2(\coth\frac{\hbar\omega}{2k_B T})}(1+\alpha^2) = \frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2k_B T} = \mathscr{E}_{Pl.}, \tag{17}$$

where $\mathscr{E}_{Pl.}$ is defined by Planck formula (11). This means that the average energy of the quantum oscillator at $T \neq 0$ has the thermodynamic meaning of its internal energy in the case of equilibrium with the ”thermal” QHB. As $T \to 0$, it passes to a similar quantity corresponding to equilibrium with the ”cold” QHB.

2 New fundamental operator - Schroedingerian

2.1 Schroedinger uncertainties relation

Because the original statement of the $\hbar kD$ is the idea of the holistic stochastic action of the QHB on the object, we introduce a new operator in the Hilbert space of microstates to implement it.

We recall the general expression of Schwartz inequality $|A|^2 \cdot |B|^2 \geqslant |A \cdot B|^2$.

Saturated Schroedinger uncertainties relations (SUR) coordinate-momentum following from it is:

$$(\Delta p)^2(\Delta q)^2 = |\tilde{R}_{qp}|^2 \equiv \sigma^2 + \frac{\hbar^2}{4}. \tag{18}$$

In the absent of stochastic action $\tilde{R}_{qp} \equiv 0$. As leading considerations, hereinafter we use an analysis of the right-hand side of the saturated SUR coordinate-momentum.

2.2 The stochastic action operator (Schroedingerian)

For not only a quantum oscillator in a QHB but also any object, the complex quantity in the right-hand side of (23)

$$\widetilde{R}_{pq} = \langle\Delta p|\Delta q\rangle \qquad \text{or } \widetilde{R}_{pq} = \langle\,|\Delta\widehat{p}\Delta\widehat{q}\,|\,\rangle \tag{19}$$

has a double meaning. On one hand, it is the amplitude of the transition from the state $|\Delta q\rangle$ to the state $|\Delta p\rangle$; on the other hand, it can be treated as the Schroedinger quantum correlator calculated over an arbitrary state $|\,\rangle$ of some operator.

As is well known, the nonzero value of quantity (24) is the fundamental attribute of nonclassical theory in which the environmental stochastic action on an object plays a significant role. Therefore, it is quite natural to assume that the averaged operator in formula (24) has a fundamental meaning. In view of dimensional considerations, we call it the stochastic action operator or Schroedingerian,

$$\widehat{j} \equiv \Delta\widehat{p}\Delta\widehat{q}. \tag{20}$$

Of course, it should be remembered that the operators $\Delta\hat{q}$ and $\Delta\hat{p}$ do not commute and their product is a non-Hermitian operator.

To analyze further, following Schroedinger, we can write the given operator

$$\widehat{j} = \frac{1}{2}\{\Delta\widehat{p}\Delta\widehat{q} + \Delta\widehat{q}\Delta\widehat{p}\} + \frac{1}{2}[\Delta\widehat{p}\Delta\widehat{q} - \Delta\widehat{q}\Delta\widehat{p}] = \widehat{\sigma} - i\,\widehat{j}_0. \tag{21}$$

It allows separating the Hermitian part of $\hat{j}$ from the anti-Hermitian one. Then the Hermitian operators $\widehat{\sigma}$ and $\hat{j}_0$ have the form

$$\widehat{\sigma} \equiv \frac{1}{2}\{\Delta\hat{p},\, \Delta\hat{q}\}; \quad \hat{j}_0 \equiv \frac{i}{2}\,[\hat{p}\,,\hat{q}] \;\; = \frac{\hbar}{2}\,\hat{I}. \tag{22}$$

It is easy to see that the mean $\sigma = \langle|\widehat{\sigma}|\rangle$ of the operator $\hat{\sigma}$ resembles the expression for the standard correlator of coordinate and momentum fluctuations in classical probability theory. It transforms into this expression if the operators $\Delta\hat{q}$ and $\Delta\hat{p}$ are replaced with c-numbers. It reflects the contribution to the transition amplitude $\tilde{R}_{pq}$ of the environmental stochastic action. Therefore, we call the operator $\hat{\sigma}$ the external action operator in what follows.

At the same time, the operators $\hat{j}_0$ and $\hat{j}$ were not previously introduced. The operator $\hat{j}_0$ of form (27) reflects a specific peculiarity of the objects to be ”sensitive” to the minimum stochastic action of the ”cold” vacuum and to respond to it adequately regardless of their states. Therefore, it should be treated as a minimum stochastic action operator. Its mean $J_0 = \frac{\hbar}{2}$ is independent of the choice of the state over which the averaging is performed, and it hence has the meaning of the invariant eigenvalue of the operator $\hat{j}_0$.

3 Effective action as a fundamental generative macroparameter

3.1 The mean of the operator $\widehat{j}$

We now construct the macrodescription of objects using their microdescription in the $\hbar kD$. It is easy to see that the mean $\tilde{J}$ of the operator $\hat{j}$ of form (26) coincides with the complex transition amplitude $\widetilde{R}_{pq}$ or Schroedinger's correlator and, in thermal equilibrium, can be expressed as

$$\tilde{J} = \langle \Psi_T(q) | \,\widehat{j}\, | \Psi_T(q) \rangle = \sigma - i J_0, \tag{23}$$

where σ and J_0 are the means of the corresponding operators. In what follows, we regard the modulus of the complex quantity $\widetilde{J}$,

$$|\tilde{J}| = \sqrt{\sigma^2 + J_0^2} = \sqrt{\sigma^2 + \frac{\hbar^2}{4}} \equiv J_{ef.} \tag{24}$$

as a new macroparameter and call it the effective action. It has the form

$$J_{ef.} = \frac{\hbar}{2} \coth \frac{\hbar\omega}{2k_B T} = \frac{U}{\omega} = \frac{k_B T_{ef}}{\omega} \tag{25}$$

for the quantum oscillator and coincides with a similar quantity previously postulated from intuitive considerations.

3.2 Effective entropy in the $\hbar kD$

The possibility of introducing entropy in the $\hbar kD$ is also based on using the wave function instead of the density operator. Using the dimensionless expressions for $\rho(q) = |\Psi(q)|^2$ and $\rho(p) = |\Psi(p)|^2$, we propose to define a formal coordinate - momentum entropy S_{qp} by the equality

$$S_{qp} = -k_B \left\{ \int \tilde{\rho}(\tilde{q}) \ln \tilde{\rho}(\tilde{q}) d\tilde{q} + \int \tilde{\rho}(\tilde{p}) \ln \tilde{\rho}(\tilde{p}) d\tilde{p} \right\}. \tag{26}$$

Substituting the corresponding expressions for $\tilde{\rho}(\tilde{q})$ and $\tilde{\rho}(\tilde{p})$ in (41), we obtain

$$S_{qp} = k_B \left\{ (1 + \ln \frac{2\pi}{\delta}) + \ln \coth \frac{\hbar\omega}{2k_B T} \right\}. \tag{27}$$

Obviously, the final result depends on the choice of the constant δ.

Choosing $\delta = 2\pi$, we can interpret expression (42) as the quantum-thermal entropy or, briefly, the $QT-$ entropy S_{QT} because it coincides exactly with the effective entropy $S_{ef.}$ obtained earlier by us in the macrotheory framework.

$$S_{QT} \equiv S_{ef.} = k_B \left\{ 1 + \ln \frac{J_{ef.}}{J_0} \right\} = k_B \left\{ 1 + \ln \Omega \right\}, \qquad (28)$$

where Ω according to Boltzmann is a number of microstates in the given macrostate. This ensures the consistency between the main results of our proposed micro- and macrodescriptions and their correspondence to experiments.

3.3 Quantum Statistical Thermodynamics on the base of the effective action

The above presentation shows that using the $\hbar kD$ developed here, we can introduce the effective action $J_{ef.}$ as a new fundamental macroparameter. The advantage of this quantity is that it has a microscopic preimage, namely, the stochastic action operator $\hat{j}$, or Schroedingerian, which has an obvious physical sense. Moreover, we can in principle express the main thermodynamic characteristics of objects in thermal equilibrium in terms of it. As is well known, temperature and entropy are the most fundamental of them.

If the notion of effective action is used, these heuristic considerations can acquire an obvious meaning. For this, we turn to expression (34) for $T_{ef.} \sim J_{ef.}$. It follows from it that the effective action is also an *intensive* macroparameter characterizing the stochastic action of the ”thermal” QHB.

In view of this, the Zero Law of equilibrium quantum Statistical Thermodynamics can be rewritten as

$$J_{ef.} = J_{ef.}^{therm.} \pm \Delta J_{ef.}, \qquad (29)$$

where $J_{ef.}^{term.}$ is effective action of QHB, $J_{ef.}$ and $\Delta J_{ef.}$ are the means of the effective action of an object and the standard deviation from it. The state of thermal equilibrium can actually be described in the sense of Newton, assuming that ”the stochastic action is equal to the stochastic counteraction” in such cases.

4 Connection with experiment

Nowadays there is alot of papers on the theory of nearly perfect fluids where the ratio of shift viscosity to entropy volume density is the subject of interest. It was shown by us that this quantity can be given by

$$\frac{J^{ef.}}{S^{ef.}} = \frac{J^{ef.}_{min}}{S^{ef.}_{min}} \cdot \frac{\coth T^{ef.}_{min}/T}{1+\ln\coth T^{ef.}_{min}/T} = \varkappa \cdot \frac{\coth \varkappa\omega/T}{1+\ln\coth \varkappa\omega/T} \to \varkappa. \tag{30}$$

In this expression,

$$\varkappa \equiv \frac{J^{ef.}_{min}}{S^{ef.}_{min}} = \frac{\hbar}{2k_B} = 3,82 \cdot 10^{-12}\text{K c.} \tag{31}$$

is the limiting ratio for $T \ll T^{ef.}$.

In our opinion, the quality is not only the notation for one of the possible combinations of the world constants $\hbar$ and k_B. It also has its intrinsic physical meaning. We sure that the quantity $\varkappa$ plays the role of a constant essentially characterizing the holistic stochastic action on the object.

The analogical with (51) relation in QSM, in contrast to one in $\hbar kD$, has the form

$$\frac{J}{S} \to \frac{\hbar\exp(-\hbar\omega/k_BT)}{k_B(\hbar\omega/k_BT)\exp(-\hbar\omega/k_BT)} = \frac{T}{\omega} \to 0. \tag{32}$$

Therefore it is now possible to compare two theories ($\hbar kD$ and QSM) experimentally by measuring the limiting value of this ratio.

The first indication that the quantity $\varkappa$ plays an important role was obtained in Andronikashvili's experiments (1948) on the viscosity of liquid Helium below the $\lambda-$ point. There are also another areas of Physics where the constant $\varkappa$ appears. Now the constant $\varkappa$ is also observed in cold atomic gases and different solids, including graphene, as a characteristic of the fundamentally new state of matter - nearly perfect fluids (Shaefer, Teaney, 2009). We hope that $\hbar kD$ can serve as an initial microtheory for constructing of modified thermodynamics fit for very small objects at ultra-low temperatures.

This work was supported by the Russian Foundation for Basic Research (project No. 10-01-90408)

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